Stanford Continuing Studies, Course MATH 07, Fall 2005

# Three views of mathematics

# LECTURE 1

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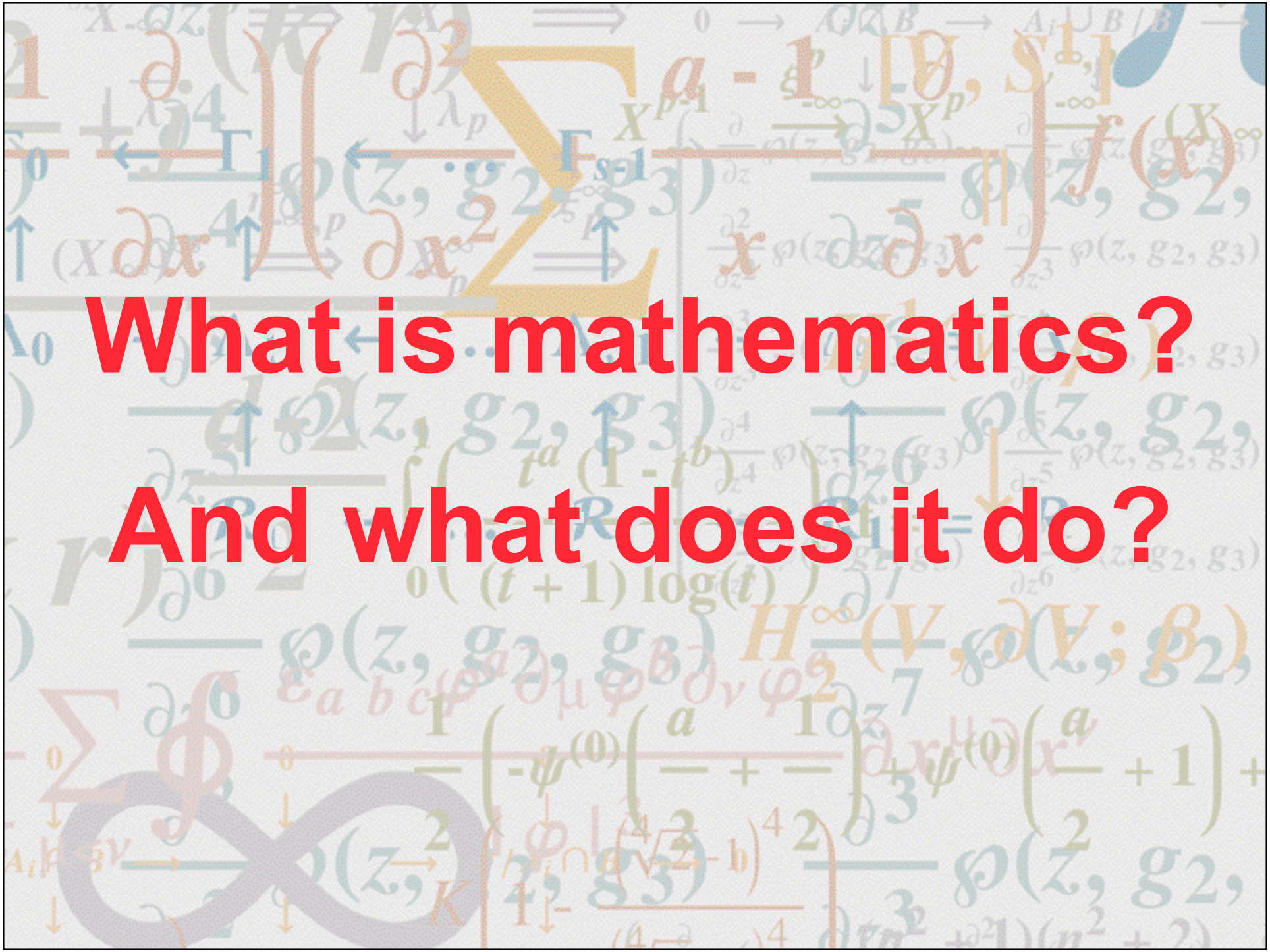

What is mathematics?
And what does it do?

# The evolution of mathematics

- **Pre-500 BC: The study of number.**
- **500 BC–300 AD: the study of number and the study of shape.**
- **17th Century: the study of number, shape, motion (calculus), and space.**
- **19th Century: the study of number, shape, motion, and space, and of the mathematical tools that are used in this study.**

# What is mathematics today?

**The science of patterns**

**alternatively**

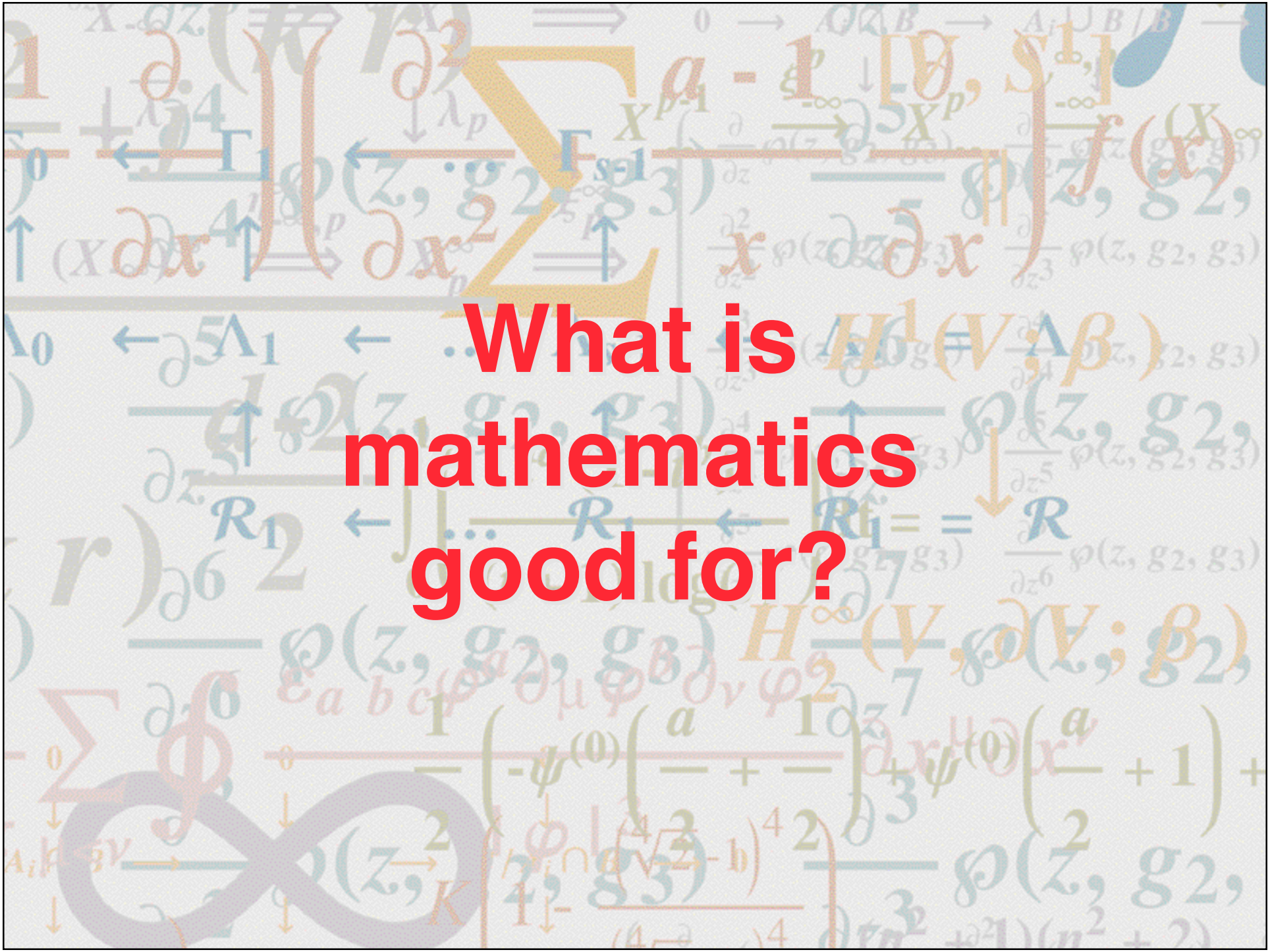
**The study of abstract structure**

# Kinds of pattern

- Counting (numbers, arithmetic)
- Numbers (number theory)
- Shape (geometry)
- Measuring (e.g. trigonometry)
- Motion and change (calculus)
- Putting things together (algebra)
- Symmetry (group theory)

# Kinds of pattern

- Position and closeness (topology)
- Reasoning (logic)
- Dynamical systems (dynamical systems theory)
- Repetitions of chance events (probability theory)
- Patterns of real world data (statistical theory)


What is mathematics good for?

# Mathematics is the language of science and technology

Galileo (1564–1642) wrote:

"To understand the universe, you have to understand the language in which it is written. And that language is mathematics."

# What is this good for?

$$\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v}\cdot\nabla\mathbf{v} = \mathbf{F} - \frac{1}{\rho}\nabla p \quad \text{Euler's equation}$$

$\mathbf{v} = -\nabla\phi$ so $\nabla \times \mathbf{v} = 0$ irrotational

$\mathbf{F} = -\nabla\Omega$ conservative

$\rho$ = const. or f(p) incompressible

$$\frac{\partial}{\partial t}\left(-\nabla\phi\right) + \nabla\phi\cdot\nabla\nabla\phi = -\nabla\Omega - \frac{1}{\rho}\nabla p$$

$$\nabla\left[-\frac{\partial\phi}{\partial t} + \frac{v^2}{2} + \Omega + \frac{p}{\rho}\right] = 0$$

$$-\frac{\partial\phi}{\partial t} + \frac{v^2}{2} + \Omega + \frac{p}{\rho} = C$$

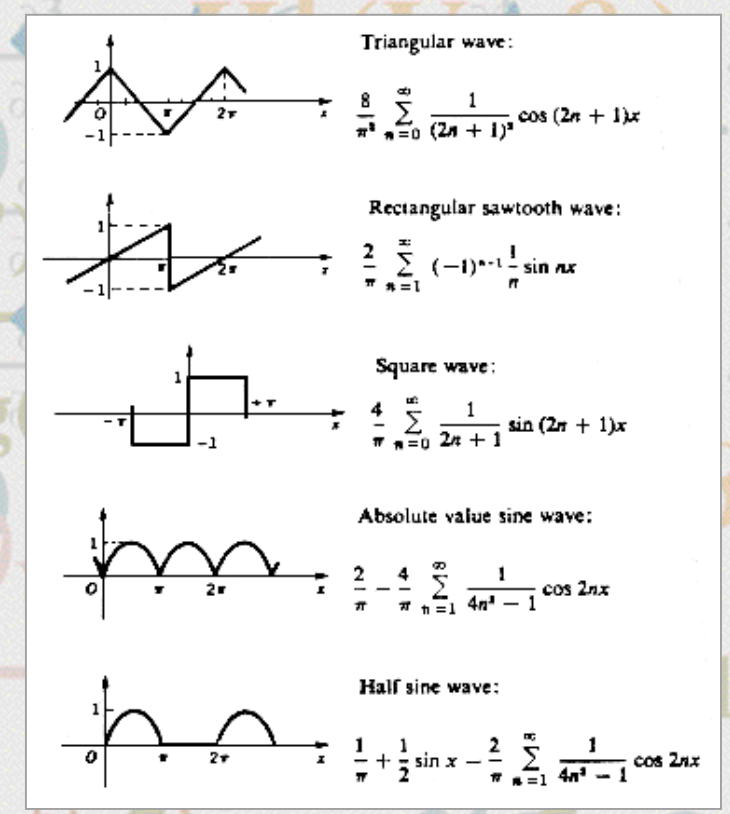
$$\frac{v^2}{2} + \Omega + \frac{p}{\rho} = C \quad \text{Bernoulli's equation}$$

**Bernoulli's Equation**

# It helps to keeps this in the air

# How does this store music?

# Using this mathematics


**Fourier series**


Mathematics
is
Everywhere

# The new enabling technology

**from *Mathematical Sciences: Adding to Australia*, January 1996:**

**"The Mathematical Sciences are critical to … economic competitiveness and quality of life, and will become more so."**

**"The Mathematical Sciences are generic and enabling technologies. They are essential to the prosperity of many value-adding industries …"**

**"…Within any particular industry sector, the Mathematical Sciences are used at all stages of a value-added chain …."**

# The currency markets

**from *Research Report for the National Academy of Science and Technology* in the Philippines, 1998:**

**"The armies of George Soros and other global investment managers use very sophisticated mathematics and computer modeling to make the financial decisions that decide the rise and fall of the peso and other currencies."**

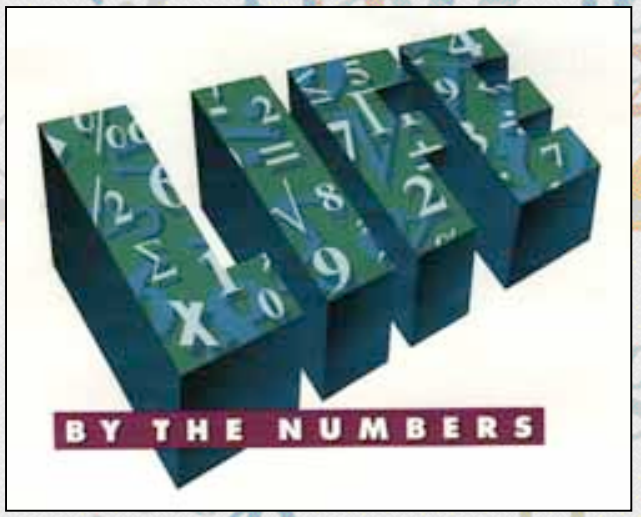
# The essential skill-base

**Alan Greenspan, Chairman, Board of Governors, U.S. Federal Reserve:**

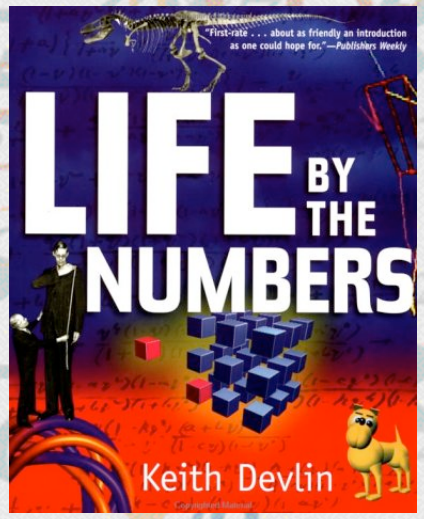
**"... in today's economy, it is becoming evident that a significant upgrading or activation of underutilized intellectual skills will be necessary to effectively engage the newer technologies. . . . In my experience, competency in mathematics – both in numerical manipulation and in understanding its conceptual foundations – enhances a person's ability to handle the more ambiguous and qualitative relationships that dominate our day-to-day decisionmaking."**

**Testimony before the Committee on Education and the Workforce U.S. House of Representatives, September 21, 2000.**

# A good resource








# *Life by the Numbers*

**PBS: WQED-tv (Pittsburgh), 1998**

**Videos available from www.montereymedia.com/science**

**Companion book published by John Wiley.**

# The fuel of the information age

**Graciela Chichilnisky, Economist, Columbia University:**

**"All of the very dynamic sunrise sectors [of the world's economies] are merging between them-selves and altering themselves through the use of mathematical tools."**

**"Mathematics works for today's society like the fossil fuels worked for the industrial society. Today, to get energy, we don't burn fossil fuels. Now, to get knowledge, we use mathematics."**

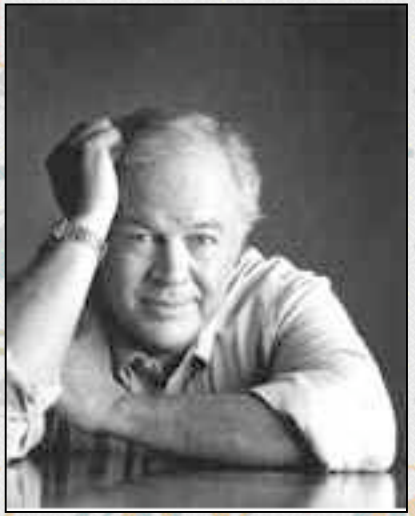
# The key to Olympic gold


"Great coaching and good intentions are not worth anything unless the goal is correct. The mathematical analysis is like turning on a light in a dark room."

Kathy Casey, U.S. Olympic figure skating coach

# The movie industry

"Mathematics is a sort of hidden tool for me. In the world of computer graphics, mathematics is behind everything I do."



Doug Trumbull, moviemaker (*2001*, *Blade Runner*, etc.)